# Optimization & Physics
# Exercise 8



Ground-state energy distributions of disordered systems can be calculated to very high precision using importance sampling. In this exercise the method is illustrated with the Sherrington-Kirkpatrick model.

## Problem 8.1 Ground-state energy distribution of the Sherrington-Kirkpatrick model

The Sherrington-Kirkpatrick model is described by the Hamiltonian:

$$\mathcal{H} = -\sum_{i<j} J_{ij} S_i S_j, \tag{1}$$

where the $S_i$ are Ising spins, and the bonds $\{J_{ij}\}$ are Gaussian distributed random variables with zero mean and standard deviation $(N-1)^{-1/2}$.
We want to calculate its ground-state distribution $P(E)$ by an importance-sampling Monte Carlo algorithm in the disorder. Similar to the Wang-Landau algorithm, the idea is to choose new bond configurations such that the ground-state energies are sampled with equal probability.
As it has been discussed in the lecture, a modified Gumbel distribution $G_{\mu,\nu,m}$ seems to fit the ground-state-distribution well and is given by

$$G_{\mu,\nu,m}(E) \sim \exp\left[m\frac{E-\mu}{\nu} - m\exp\left(\frac{E-\mu}{\nu}\right)\right]. \tag{2}$$

Usually one has to estimate the parameters by doing a simple-sampling Monte Carlo simulation first, but for the Sherrington-Kirkpatrick model an estimate for these parameters has been done by Koerner et al. and we will look at the choice $\mu = -10.429$, $\nu = 3.233$ and $m = 8$ for $N = 16$ spins.
The procedure of the algorithm is as follows:
Start with a choice of bonds $J_{\text{start}} = \{J_{ij}\}$ and calculate the corresponding ground-state energy with one of the many methods, which have been presented in this course. Choose a site at random and change all the bonds connected to this site by assigning new random variables to them. Calculate the new ground-state energy and accept the new configuration with probability

$$P = \min\{\frac{G_{\mu,\nu,m}(E(J_{\text{old}}))}{G_{\mu,\nu,m}(E(J_{\text{new}}))}, 1\}. \tag{3}$$

While repeating these steps, you are required to bin the visited ground-state energies. With the help of the modified Gumbel function you are then able to conclude from your histogram the actual ground-state distribution $P(E)$ of the Sherrington-Kirkpatrick model.