Optimization & Physics
Exercise 5

HS 07
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The goal of this exercise is to practice the basics of graph theory and to convince oneself of its utility on optimization problems.

Problem 5.1 Eulerian graphs

A graph is an *Euler graph*, or *Eulerian* if there is a simple cycle v_0, e_0, v_1, e_1, ..., v_n, e_n where $v_0 = v_n$ and every edge is traversed exactly once. Show, that a graph G is Eulerian if and only if all vertices have even degree.

Problem 5.2 Directed polymers in a random medium

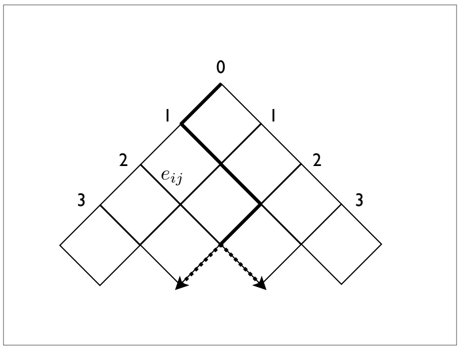

Abbildung 1: The bold line is the upper part of a polymer, which starts at the $(0, 0)$ vertex. Its configuration is given by the set of $\{x_{ij}\}$. The random nature of the medium is described by the random potential e_{ij}.

We will model a polymer in a random medium by searching for a directed optimal path on the links of a lattice. The polymer is represented as a line, which passes through links (ij) connecting vertices i and j. The weights of the edges correspond to random potential energies e_{ij} and the Hamiltonian can be written as:

$$H = \sum_{(ij)} e_{ij} x_{ij} \tag{1}$$

where the sum is over edges (ij) of the graph and x_{ij} represents the polymer configuration. We set $x_{ij} = 1$ if the polymer passes through the edge (ij) and $x_{ij} = 0$ otherwise.
Interpreting the energies e_{ij} as distances and the lattice as a directed graph, the search for an optimal polymer configuration in a random medium becomes a shortest path problem, which can be solved with Dijkstra's algorithm. Since distances are positive, make sure you shift the energies in order that they become positive. Use Dijkstra's algorithm to find an optimal polymer configuration for a given random landscape, i.e., a given choice of the set $\{e_{ij}\}$.